Math 3070/6070 Homework 4
Due: Oct 28th, 2022

1. (2.13) Consider a sequence of independent coin flips, each of which has probability p of being heads. Define a random variable X as the length of the run (of either heads or tails) started by the first trial. (For example, $X = 3$ if either TTTH or HHHT is observed.) Find the distribution of X and find EX.

2. (2.15) Suppose the pdf $f_X(x)$ of a random variable X is an *even* function. ($f_X(x)$ is an even function if $f_X(x) = f_X(-x)$ for every x.) Show that

    1. X and $-X$ are identically distributed.

    2. $M_X(t)$ is symmetric about 0.

3. (2.33) In each of the following cases verify the expression given for the moment generating function, and in each case use the mgf to calculate EX and $Var(X)$.

    1. $\Pr(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$, $M_X(t) = e^{\lambda(e^t-1)}$, $x = 0, 1, \ldots; \lambda > 0$.
    2. $\Pr(X = x) = p(1-p)^x$, $M_X(t) = \frac{p}{1-(1-p)e^t}$, $x = 0, 1, \ldots; 0 < p < 1$.
    3. $f_X(x) = \frac{e^{-(x-\mu)^2/(2\sigma^2)}}{\sqrt{2\pi}\sigma}$, $M_X(t) = e^{\mu t + \sigma^2 t^2/2}$, $-\infty < x < \infty; -\infty < \mu < \infty, \sigma > 0$.

4. (3.3) The flow of traffic at certain street corners can sometimes be modeled as a sequence of Bernoulli trials by assuming that the probability of a car passing during any given second is a constant p and that there is no interaction between the passing of cars at different seconds. If we treat seconds as indivisible time units (trials), the Bernoulli model applies. Suppose a pedestrian can cross the street only if no car is to pass during the next 3 seconds. Find the probability that the pedestrian has to wait for exactly 4 seconds before starting to cross.

5. (3.12) Suppose X has a $Binomial(n, p)$ distribution and let Y have a negative binaomial(r, p) distribution. Show that $F_X(r-1) = 1 - F_Y(n-r)$.