

# Math 3070/6070 Homework 3

Due: Oct 14th, 2022

1. (2.1) In each of the following find the pdf of  $Y$ . Show that the pdf integrates to 1.

1.  $Y = X^3$  and  $f_X(x) = 42x^5(1-x), 0 < x < 1$

2.  $Y = 4X + 3$  and  $f_X(x) = 7e^{-7x}, 0 < x < \infty$

3.  $Y = X^2$  and  $f_X(x) = 30x^2(1-x)^2, 0 < x < 1$

2. (2.2) In each of the following find the pdf of  $Y$

1.  $Y = X^2$  and  $f_X(x) = 1, 0 < x < 1$

2.  $Y = -\log(X)$  and  $X$  has pdf

$$f_X(x) = \frac{(n+m+1)!}{n!m!} x^n (1-x)^m, \quad 0 < x < 1, \quad m, n \text{ positive integers}$$

3.  $Y = e^X$  and  $X$  has pdf

$$f_X(x) = \frac{1}{\sigma^2} x e^{-(x/\sigma)^2/2}, \quad 0 < x < \infty, \quad \sigma^2 \text{ a positive constant}$$

3. (2.4) Let  $\lambda$  be a fixed positive constant, and define the function  $f(x)$  by  $f(x) = \frac{1}{2}\lambda e^{-\lambda x}$  if  $x \geq 0$  and  $f(x) = \frac{1}{2}\lambda e^{\lambda x}$  if  $x < 0$ .

1. Verify that  $f(x)$  is a pdf.

2. If  $X$  is a random variable with pdf given by  $f(x)$ , find  $\Pr(X < t)$  for all  $t$ . Evaluate all integrals.

3. Find  $\Pr(|X| < t)$  for all  $t$ . Evaluate all integrals.

4. (2.6) In each of the following find the pdf of  $Y$  and show that the pdf integrates to 1.

1.  $f_X(x) = \frac{1}{2}e^{-|x|}, -\infty < x < \infty; Y = |X|^3$

2.  $f_X(x) = \frac{3}{8}(x+1)^2, -1 < x < 1; Y = 1 - X^2$

3.  $f_X(x) = \frac{3}{8}(x+1)^2, -1 < x < 1; Y = 1 - X^2$  if  $X \leq 0$  and  $Y = 1 - X$  if  $X > 0$

5. (2.9) If the random variable  $X$  has pdf

$$f(x) = \begin{cases} \frac{x-1}{2}, & 1 < x < 3 \\ 0, & \text{otherwise,} \end{cases}$$

find a monotone function  $u(x)$  such that the random variable  $Y = u(X)$  has a *uniform*(0,1) distribution.