

# Math 3070/6070 Homework 2

Due: Sept 22nd, 2022

1. (1.47 a - d) Prove that the following functions are cdfs.

1.  $\frac{1}{2} + \frac{1}{\pi} \tan^{-1}(x), x \in (-\infty, \infty)$

2.  $(1 + e^{-x})^{-1}, x \in (-\infty, \infty)$

3.  $e^{-e^{-x}}, x \in (-\infty, \infty)$

4.  $1 - e^{-x}, x \in (0, \infty)$

2. (1.51) An appliance store receives a shipment of 30 microwave ovens, 5 of which are (unknown to the manager) defective. The store manager selects 4 ovens at random, without replacement, and tests to see if they are defective. Let  $X$  = number of defectives found. Calculate the pmf and cdf of  $X$  and plot the cdf.

3. (1.52) Let  $X$  be a continuous random variable with pdf  $f(x)$  and cdf  $F(x)$ . For a fixed number  $x_0$ , define the function

$$g(x) = \begin{cases} f(x)/[1 - F(x_0)] & x \geq x_0 \\ 0 & x < x_0. \end{cases}$$

Prove that  $g(x)$  is a pdf. (Assume that  $F(x_0) < 1$ .)

4. (1.53) A certain river floods every year. Suppose that the low-water mark is set at 1 and the high-water mark  $Y$  has distribution function

$$F_Y(y) = \Pr(Y \leq y) = 1 - \frac{1}{y^2}, \quad 1 \leq y < \infty.$$

1. Verify that  $F_Y(y)$  is a cdf.
  2. Find  $f_Y(y)$ , the pdf of  $Y$ .
  3. If the low-water mark is reset at 0 and we use a unit of measurement that is  $\frac{1}{10}$  of that given previously, the high-water mark becomes  $Z = 10(Y - 1)$ . Find  $F_Z(z)$ .
5. (1.54) For each of the following, determine the value of  $c$  that makes  $f(x)$  a pdf.

1.  $f(x) = c \sin x, \quad 0 < x < \pi/2$

2.  $f(x) = ce^{-|x|}, \quad -\infty < x < \infty$